

General relativity extended to non-Riemannian space-time geometry.

Yuri A. Rylov

Institute for Problems in Mechanics, Russian Academy of Sciences,
101-1, Vernadskii Ave., Moscow, 119526, Russia.

e-mail: rylov@ipmnet.ru

Web site: <http://rsfq1.physics.sunysb.edu/~rylov/yrylov.htm>
or mirror Web site: <http://gasdyn-ipm.ipmnet.ru/~rylov/yrylov.htm>

Abstract

The gravitation equations of the general relativity, written for Riemannian space-time geometry, are extended to the case of arbitrary (non-Riemannian) space-time geometry. The obtained equations are written in terms of the world function in the coordinateless form. These equations determine directly the world function, (but not only the metric tensor). As a result the space-time geometry appears to be non-Riemannian. Invariant form of the obtained equations admits one to exclude influence of the coordinate system on solutions of dynamic equations. Anybody, who trusts in the general relativity, is to accept the extended general relativity, because *the extended theory does not use any new hypotheses. It corrects only inconsequences and restrictions of the conventional conception of general relativity.* The extended general relativity predicts an induced antigravitation, which eliminates existence of black holes.

1 Introduction

In this paper we consider dynamic equations for the gravitational field which are obtained at the generalization of the relativity theory on the case of the most general space-time geometry. The general relativity supposes, that the Riemannian geometry is the most general space-time geometry. This supposition is based on our insufficient knowledge of a geometry, when one supposes that any geometry is axiomatizable. It means, that any geometry is constructed as a logical construction. In reality there exist nonaxiomatizable space-time geometries [1], which are constructed by means of the deformation principle [2] as a deformation of the proper Euclidean geometry. This geometry is described completely by the world function [3] and only by the world function. Such a geometry is called a physical geometry,

because physicists need such a geometry, which is a science on location of geometrical objects and on their shape, (but not as a logical construction). Physicists use a geometry as a tool for investigation of the space-time properties. Physicists are indifferent to the question, whether or not a geometry is a logical construction.

Physical geometry may be continuous, or discrete. It may be even granular, i.e. partly continuous and partly discrete. The physical geometry is described by the same manner in all cases. Properties of the physical geometry are determined only by properties of the world function (but not by properties of the point set, where the geometry is given). As a result the physical geometry may be formulated in the coordinateless form (only in terms of the world function). A good illustration of this fact is the following example.

Let the proper Euclidean geometry be given in the Cartesian coordinates (x, y) on the square $[0, 1] \times [0, 1]$. It means, that the world function is given on this square. Let us map the square $[0, 1] \times [0, 1]$ onto the one-dimensional segment $[0, 1]$. Let the mapping $(x, y) \rightarrow X$ be one-to-one. It is possible only, if the mapping is discontinuous at any point. For instance, the mapping can be realized as follows. Let coordinates x, y, X be presented in the form of decimal fractions

$$x = 0.\alpha_1\alpha_2\alpha_3\dots, \quad y = 0.\beta_1\beta_2\beta_3\dots, \quad X = 0.\alpha_1\beta_1\alpha_2\beta_2\alpha_3\beta_3\dots \quad (1.1)$$

where α and β are decimal ciphers. The formulas (1.1) realizes one-to-one mapping $(x, y) \leftrightarrow X$. Now the world function σ is given on one-dimensional segment $[0, 1]$.

$$\sigma(X_1; X_2) = \sigma(x_1, y_1; x_2, y_2)$$

Nevertheless, considering the world function on the one-dimensional segment $[0, 1]$, one can reconstruct the proper Euclidean geometry. In particular, one can determine, that the geometry on the segment $[0, 1]$ is the two-dimensional Euclidean geometry (in the sense, that the maximal number of linear independent vectors is equal two), although the geometry is given on one-dimensional segment.

For construction of a physical geometry it is sufficient to give a world function for any pair of points of the point set, where the geometry is given. One does not need to prove numerous theorems and to test a compatibility of geometric axioms. The world function $\sigma(P, Q) = \frac{1}{2}\rho^2(P, Q)$ is a function of two points P and Q , where $\rho(P, Q)$ is a distance between the two points. The number of possible world functions is much more, than the number of infinitesimal intervals $dS^2 = g_{ik}(x) dx^i dx^k$, which are functions of only one point. In particular, there is only one isotropic uniform geometry (the geometry of Minkowski) in the set of Riemannian geometries. It is described by the world function σ_M . In the set of physical geometries any geometry is isotropic and uniform, if it is described by the world function $\sigma = F(\sigma_M)$, where F is an arbitrary function, having the property $F(0) = 0$ and σ_M is the world function of the geometry of Minkowski.

In particular, the space-time geometry, described by the world function

$$\sigma = \sigma(\sigma_M) = \sigma_M + \lambda_0^2 \text{sgn}(\sigma_M), \quad \lambda_0^2 = \frac{\hbar}{2bc} = \text{const} \quad (1.2)$$

is uniform and isotropic. Here \hbar is the quantum constant, c is the speed of the light and b is some universal constant. Besides, this geometry is nonaxiomatizable and discrete. In this space-time geometry any motion of a free pointlike particles is multivariant (stochastic). Statistical description of this stochastic motion is equivalent to the quantum description in terms of the Schrödinger equation [4]. This circumstance admits one to obtain a statistical foundation of quantum mechanics and to interpret quantum effects as geometrical effects. It admits one to exclude the quantum principles from the set of prime physical principles and to reduce the number of physical essences, what is important for fundamental physical theories.

By definition the special relativity is a consideration of physical phenomena in the flat uniform isotropic space-time. In the set of Riemannian geometries there is only one such a geometry. It is the space-time geometry of Minkowski. Description of physical phenomena in the geometry (1.2) should be qualified as an extended special relativity, because the space-time geometry (1.2) is isotropic and uniform, but it is non-Riemannian.

Statistical foundation of quantum theory shows also, that the real space-time geometry may be non-Riemannian, and one cannot restrict oneself, considering only the Riemannian space-time geometries.

Generalization of the general relativity on the case of physical space-time geometry appears to be possible only at taking into account two essential clauses:

1. Consideration of physical geometries.
2. Use of adequate relativistic concepts, and, in particular, a use of the relativistic concept of the events nearness.

The reasons of violation of the first condition are investigated in [5].

In the beginning of the twentieth century the theoretical physics developed on the way of geometrization. The special relativity and the general relativity were only stages of this geometrization. But the physics geometrization appeared to be restricted by our poor knowledge of geometry, when one knew only axiomatizable geometries. One was not able to work with discrete geometries and geometries with restricted divisibility. Quantum effects might be explained easily by multivariance of the space-time geometry. However, the property of multivariance was not known in the beginning of the twentieth century, and scientists were forced to introduce new (quantum) principles of dynamics. As a result the quantum paradigm of the microcosm physics development appeared. The quantum paradigm dominated during the whole twentieth century. The quantum paradigm contains more essences, than it is necessary.

In the end of the twentieth century, when our knowledge of geometry became more complete, we may return to the program of further geometrization of physics. The geometrical paradigm appeared to be possible, when, using classical dynamic principles, quantum effects are freely explained by the properties of the space-time geometry. The geometrical paradigm is more attractive, because it uses less essences,

than the quantum paradigm does. To replace the quantum paradigm by the geometrical one, it is necessary to generalize the special relativity and the general relativity on the case of an arbitrary physical geometry of space-time.

The physical geometry is a geometry, which is described completely by the world function, (world function is a half of the squared distance). Practically, the physical geometry is a metric geometry, which is liberated from all constraints on metric except the constraint, that the world function (or metric) is equal to zero for two coinciding points. The physical geometry is a very simple construction [1, 2]. For constructing the physical geometry one does not need to formulate axioms and to prove numerous theorems. It is sufficient to know the proper Euclidean geometry, which is used as a standard physical geometry. All definitions of the proper Euclidean geometry \mathcal{G}_E may be formulated in terms of the Euclidean world function σ_E . Replacing the Euclidean world function σ_E in all definitions of the Euclidean geometry by other world function σ , one obtains all definitions of the physical geometry \mathcal{G} , described by the world function σ .

Besides, the physical geometry is a monistic conception, which is described by the only fundamental quantity σ . All other geometrical quantities and concepts are expressed via fundamental quantity automatically. This circumstance admits one to modify a physical geometry easily, because all other geometric quantities concepts are modified automatically at modification of the fundamental quantity σ . [6]. Program of physics geometrization admits one to construct a monistic conception of physics with the fundamental quantity σ .

The set of all Riemannian geometries is only a small part of the set of all physical geometries. A generalization of the relativity theory on the case of arbitrary physical geometry admits one to obtain such results, which cannot be obtained in the framework of the Riemannian geometry. The generalization of the special relativity (motion of particle in the given space-time geometry) on the case of arbitrary physical space-time geometry has been made already [7]. A generalization of description of the matter influence on the arbitrary space-time geometry met some problems. These problems are connected with the fact, that in the relativity theory some basic concepts are taken from the nonrelativistic physics. Concepts of nonrelativistic physics are inadequate for consecutive geometric description of the relativity theory and for generalization of this description on the case of a more general space-time geometry.

Practically all physical phenomena on the Earth are nonrelativistic. At first, we study nonrelativistic physics with its nonrelativistic concepts. Relativistic effects appeared as corrections to nonrelativistic physics. In the beginning of the twentieth century the relativistic physics was presented in terms of slightly corrected nonrelativistic concepts. For instance, the relativity principle has been presented as invariance of the dynamic equations with respect to Lorentz transformation and as existence of the supreme speed of the interaction propagation. Such a formulation is useful for pedagogical goals, when one needs to transit from concepts of nonrelativistic physics to relativistic ones. However, such a formulation is not effective, when one tries to develop the relativistic physics. In this case one should use concepts,

which are adequate for relativistic physics. In particular, the relativity principle is formulated in adequate concepts as follows. The relativistic physics is a physics in the pseudo-Euclidean space-time geometry of index 1 (geometry of Minkowski or that of Kaluza-Klein). All other details of description are corollaries of properties of the space-time geometry. For instance, properties, concerning the role of the light speed, are pure geometrical properties of the space-time.

Unfortunately, the formulation of the relativity theory in adequate (geometrical) terms is used rare. The main difference of space-time geometry in a relativistic theory from that in the non-relativistic (Newtonian) physics is as follows. Relativistical event space (space-time) geometry is described by one invariant (space-time interval), whereas in the Newtonian physics the event space is described by two invariants (spatial distance and temporal interval). Sometimes one does not mention this difference in textbooks. Instead one speaks on difference in transformation laws. In reality, the difference in the number of invariants is a fundamental property, whereas the difference of the transformation laws is a very special property, because it is essential only for flat space-time geometry. Besides, the transformational properties are used only at description at some coordinate system. They are useless at the coordinateless description. This difference of formulations is not essential, when the theory is used for calculation of concrete physical effects. However, this difference becomes essential, if one tries to obtain a generalization of the relativity theory on the case of the arbitrary space-time geometry.

For instance, the concept of a pointlike particle as a point in the configuration space is a nonrelativistic concept. It needs concepts of velocity and acceleration of this particle. These concepts are secondary concepts, which can be introduced only after introduction of the linear vector space and, in particular of a coordinate system. These concepts are inadequate in the case of a discrete space-time geometry. As a result the concept of velocity and that of acceleration cannot be used in an extension of the relativity theory to the case of arbitrary space-time geometry, which may be discrete.

In the general relativity all interactions (electromagnetic and gravitational) are supposed to be short-range interactions. Concept of short-range interaction is based on the nonrelativistic concept of the events nearness. The events are considered as points in the event space (space-time). Two events are considered to be near, if they happen in the same place at the same time moment. This definition of nearness of events is nonrelativistic, because this definition refers to a spatial distance and to a temporal interval at once. A consistent relativistic concept of nearness is to contain a reference to only quantity: space-time interval, (or world function). For instance, if a supernew star flashed very far, and an observer on the Earth observed this flash, the event of flash and the event of this flash observation on the Earth are near (close) events.

According to common viewpoint the statement on nearness of the two events (flash and observation of this flash) seems to be rather strange and unexpected. However, from consistent relativistic viewpoint the two events are near, because space-time interval between them is equal to zero.

The problem of relativistic concept of nearness is discussed in [8]. It is known as the principle of Fokker [9], which is interpreted as a conception of the action at a distance (but not as a relativistic concept of nearness). The action at a distance is treated as a direct influence of one object onto another one without intermediate agent circulatory between them.

2 Relativistic concept of nearness

Let us consider the proper Euclidean geometry. Let $\rho(P, Q)$ be the Euclidean distance between the points P and Q . The set O_ε of points P , defined by the relation

$$O_\varepsilon = \{P \mid \rho(O, P) < \varepsilon\}, \quad \varepsilon > 0 \quad (2.1)$$

is called ε -vicinity of the point O . If the parameter ε is small, the points P and O are near ($P \simeq O$). If $\varepsilon \rightarrow 0$, ε -vicinity O_ε degenerates into one point $O_0 = O$. It is easy to see, that, if $P \simeq Q$, then $Q \simeq P$.

The relation of nearness in the proper Euclidean geometry has the property of transitivity: If $P \in O_\varepsilon$ and $Q \in O_\varepsilon$, then $P \in Q_{2\varepsilon}$ and $Q \in P_{2\varepsilon}$. It follows from the triangle axiom,

$$\rho(O, P) + \rho(O, Q) \geq \rho(Q, P)$$

which is valid for the proper Euclidean geometry. If $\varepsilon \rightarrow 0$, then $2\varepsilon \rightarrow 0$ also. It means that, if $P \simeq O$ and $Q \simeq O$, then $P \simeq Q$.

The property of transitivity seems to be a natural property of the relation of nearness. However, the transitivity property of the nearness relation does not take place in the space-time geometry, for instance, in the geometry of Minkowski. In this case the ε -vicinity O_ε of the point O is defined by the relation

$$O_\varepsilon = \{R \mid |\rho(O, R)| < \varepsilon\}, \quad \rho(O, R) = \sqrt{2\sigma_M(O, R)} \quad (2.2)$$

Here $\sigma_M(P, Q) = \sigma_M(x, x')$ is the world function of the space-time of Minkowski

$$\sigma_M(P, Q) = \sigma_M(x, x') = \frac{1}{2} (g_M)_{ik} (x^i - x'^i) (x^k - x'^k) \quad (2.3)$$

x, x' are coordinates of points P and Q in some inertial coordinate system, and $(g_M)_{ik}$ is the metric tensor in this coordinate system.

In this case the points with coordinates $P = \{\sqrt{a^2 + \varepsilon^2}, a, 0, 0\}$ and $Q = \{\sqrt{a^2 + \varepsilon^2}, -a, 0, 0\}$ belong to ε -vicinity of the point $O = \{0, 0, 0, 0\}$, whereas $P \notin Q_{2\varepsilon}$, because

$$|2\sigma(P, Q)| = |\rho^2(P, Q)| = 4a^2 \quad (2.4)$$

As far as the quantity a may be indefinitely large, the distance between the points P and Q may be very large, although both points are near to the point O ($P \simeq O$ and $Q \simeq O$).

In the space-time of Minkowski the ε -vicinity of the point $O = \{0, 0, 0, 0\}$ is a region of the space-time between two hyperboloids

$$(x^0)^2 - \mathbf{x}^2 = \varepsilon^2, \quad (x^0)^2 - \mathbf{x}^2 = -\varepsilon^2 \quad (2.5)$$

Formally the relation (2.5) determines a sphere of radius ε in the space-time of Minkowski. At $\varepsilon \rightarrow 0$ this region turns to the light cone with the vertex at the point O . Thus, at $\varepsilon \rightarrow 0$ in the proper Euclidean geometry the ε -vicinity of the point P is the same point P , whereas in the geometry of Minkowski the ε -vicinity of the point P is the light cone with the vertex at the point P .

In the nonrelativistic physics the ε -vicinity O_ε of the point $O = \{0, 0, 0, 0\}$ is defined by relations

$$O_\varepsilon = \{ \{x^0, \mathbf{x}\} \mid |x^0| < \varepsilon \wedge |\mathbf{x}| < \varepsilon \} \quad (2.6)$$

In the limit $\varepsilon \rightarrow 0$, the ε -vicinity (2.6) turns to one point O . Thus, in the non-relativistic physics there are only one near point, whereas in the relativistic physics there is a continual set of near points. This difference appears to be very important in definition of short-range interaction between particles.

Let us stress, that introducing cone-shaped ε -vicinity and nearness of points on the light cone to the vertex of the cone, **we do not suggest any hypothesis. We follow only the relativity principle.** If we follow the relativity principles, we should accept the fact of the cone-shaped ε -vicinity, because pointlike shape of the ε -vicinity in the limit $\varepsilon \rightarrow 0$ is a remnant of the nonrelativistic theory.

3 Relativistic concept of a pointlike particle

In the consecutive geometric description any particle is realized by its skeleton. In the case of a pointlike particle the skeleton is the ordered set of two points $\{P_s, P_{s+1}\}$. The vector $\mathbf{P}_s \mathbf{P}_{s+1}$ describes the geometric momentum of particle. The length $|\mathbf{P}_s \mathbf{P}_{s+1}| = \sqrt{2\sigma(P_s, P_{s+1})}$ of the vector $\mathbf{P}_s \mathbf{P}_{s+1}$ describes the geometric mass of particle. Such a description is a pure geometric one.

Motion of a pointlike particle is described by a world chain \mathcal{C} , consisting of connected links $\mathcal{T}_{[P_s P_{s+1}]}$

$$\mathcal{C} = \sum_s \mathcal{T}_{[P_s P_{s+1}]} \quad (3.1)$$

Any link $\mathcal{T}_{[P_s P_{s+1}]}$ is a segment of straight line, determined by the skeleton $\mathcal{P}_1^{(s)} = \{P_s, P_{s+1}\}$. The link $\mathcal{T}_{[P_s P_{s+1}]}$ is a set of points, determined by the relation

$$\mathcal{T}_{[P_s P_{s+1}]} = \left\{ R \mid \sqrt{2\sigma(P_s, R)} + \sqrt{2\sigma(R, P_{s+1})} - \sqrt{2\sigma(P_s, P_{s+1})} = 0 \right\} \quad (3.2)$$

The length

$$\mu = \sqrt{2\sigma(P_s, P_{s+1})} \quad (3.3)$$

of all links is the same. The length μ is the geometric mass of the particle, which is connected with the usual mass m of the particle by the relation

$$m = b\mu = b\sqrt{2\sigma(P_s, P_{s+1})} \quad (3.4)$$

where b is the same universal constant, which appears in (1.2)

Complicated (not pointlike) particles are described by a more complicated skeleton $\mathcal{P}_n = \{P_0, P_1, \dots, P_n\}$ [7].

Description of the particle motion does not need an introduction of a coordinate system. Details of such a description of the particle motion may be found in [7]. Such a description is generalized easily on the case of arbitrary space-time geometry (in particular, discrete one). In the microcosm the structure of the world chain (3.1) is essential, but outside the microcosm one may consider the length μ of a link $\mathcal{T}_{[P_s P_{s+1}]}$ to be infinitesimal, and to replace the world chain by a smooth world line.

Let \mathcal{L} be a world line of a pointlike particle, and the point $P \in \mathcal{L}$. A set \mathbb{N}_P of events Q , which are near to the point P is different from the relativistic viewpoint and from the nonrelativistic one. From nonrelativistic (conventional) viewpoint $\mathbb{N}_P = \{P\}$, whereas from the relativistic viewpoint $\mathbb{N}_P = \mathcal{C}_P$, where \mathcal{C}_P is the light cone with the vertex at the point P .

$$\mathcal{C}_P = \{R | \sigma(P, R) = 0\} \quad (3.5)$$

It is known, that the electromagnetic interaction between two pointlike charged particles is carried out only via points, connecting with vanishing space-time interval (retarding interaction), i.e. via points, which are near from the relativistic viewpoint. The same is valid for the gravitational interaction. On the other hand, the near points of the world line \mathcal{L} should be interpreted in a sense of points belonging to the world line. In this sense any interaction of two pointlike particles via near points may be interpreted as a direct interaction (collision).

The light cones with vertexes at the points $P \in \mathcal{L}$, may be considered as attributes of the pointlike particle, which is described by the world line \mathcal{L} . We shall consider these light cones, directed into the past, as bunches of isotropic straight lines \mathcal{H} . In other words, any world line \mathcal{L} of a pointlike particle is equipped by bunches \mathcal{C}_P of hair \mathcal{H}_P at any point $P \in \mathcal{L}$. Any hair \mathcal{H}_P consists of points $R \in \mathcal{H}_P$, which are near to the point $P \in \mathcal{L}$, ($\sigma(P, R) = 0$, $R \in \mathcal{H}_P$) on the world line \mathcal{L} . The point P is a footing of the hair \mathcal{H}_P . The length of the hair \mathcal{H}_P is equal to zero, because the hair \mathcal{H}_P consists of points, which are near to the point P . Although the length of any hair is equal to zero, nevertheless the hairs of any world line cover the whole space-time. When some point $P' \in \mathcal{H}_P$, $P \in \mathcal{L}_1$ of the world line \mathcal{L}_1 hair coincides with a point $P' = P_2 \in \mathcal{L}_2$ of other world line \mathcal{L}_2 , the particle \mathcal{L}_2 transfers a part of its momentum to the particle \mathcal{L}_1 . See figure

What part of its momentum does the particle \mathcal{L}_2 transfer, depends on the point $P' \in \mathcal{H}_P$, which is a common point with \mathcal{L}_2 ($P' = P_2 \in \mathcal{L}_2$).

Although the length of any part of the hair is equal to zero, nevertheless there is some invariant parameter along any hair \mathcal{H} . This parameter l_r is the relative length

of the hair segment. The relative length (r -length) of the point P is the more, the "farther" the point $R \in \mathcal{H}_P$ lies from the footing P of the hair \mathcal{H}_P . The r -length l_r of the point $R \in \mathcal{H}_P$ is defined by the relation

$$l_r = l_r(P, R) = \frac{(\mathbf{PR} \cdot \mathbf{Q}_0 \mathbf{Q}_1)}{|\mathbf{Q}_0 \mathbf{Q}_1|} \quad (3.6)$$

where vector $\mathbf{Q}_0 \mathbf{Q}_1$ is an arbitrary timelike vector ($\sigma(O_0, Q_1) > 0$). The scalar product $(\mathbf{PR} \cdot \mathbf{Q}_0 \mathbf{Q}_1)$ of vectors \mathbf{PR} and $\mathbf{Q}_0 \mathbf{Q}_1$ is defined by the relation

$$(\mathbf{PR} \cdot \mathbf{Q}_0 \mathbf{Q}_1) = \sigma(P, Q_1) + \sigma(R, Q_0) - \sigma(P, Q_0) - \sigma(R, Q_1) \quad (3.7)$$

$$|\mathbf{Q}_0 \mathbf{Q}_1| = \sqrt{(\mathbf{Q}_0 \mathbf{Q}_1 \cdot \mathbf{Q}_0 \mathbf{Q}_1)} = \sqrt{2\sigma(Q_0, Q_1)} \quad (3.8)$$

It follows from expressions (3.6) - (3.8), that the relative length is invariant, because it is expressed in terms of the world function. Numerical value of the r -length depends on the choice of the timelike vector $\mathbf{Q}_0 \mathbf{Q}_1$. Sign of the r -length depends on the choice of the timelike vector $\mathbf{Q}_0 \mathbf{Q}_1$ also. However, the order of points on the hair, directed to the past, is determined single-valuedly by the value of the r -length.

If for some choice of the timelike vector $\mathbf{Q}_0 \mathbf{Q}_1$

$$|l_r(P, R_1)| < |l_r(P, R_2)|, \quad R_1, R_2 \in \mathcal{H}_P \quad (3.9)$$

then the relation (3.9) takes place for any other choice of timelike vector $\mathbf{Q}_0 \mathbf{Q}_1$. It means that the point R_1 is located between the points P and R_2 . The quantity of the transferred momentum is inversely to the r -length $l_r(P, P')$ between the footing of the hair $P \in \mathcal{L}_1$ and the point $P' \in \mathcal{L}_2$, $P' \in \mathcal{H}_P$.

The concept of the world line hair admits one to consider and to calculate electromagnetic and gravitational interaction of particles as a direct collision of one particle with a hair of other particle. As far as the hairs of a world line are considered as attributes of a particle, one may consider electromagnetic and gravitational interaction of particles as a direct collision of particles. Such a description of the particle interaction *does not mention about gravitational and electromagnetic fields. Such a description is a consecutive relativistic description.*

In the nonrelativistic theory the electromagnetic and gravitational fields are *essences, which exist independently of the matter*. These essences provide the momentum transfer from one particle to another one. Introduction of such essences is necessary, because the nonrelativistic concept of nearness is used. In the consecutive relativistic theory, which uses relativistic concept of nearness, one does not need to consider the electromagnetic and gravitational fields as additional essences. It is sufficient to consider them as a manner of description of particle interaction. The less number of essences is contained in a fundamental theory, the more perfect fundamental theory takes place.

Our conclusion, that gravitational and electromagnetic fields are not physical essences (they are only attributes of the world function) seems rather unexpected for

most physicists. It is connected with the fact, that the relativity theory is considered usually as a correction to the nonrelativistic physics. As a result the relativity theory is presented almost in all textbooks in terms of concepts of nonrelativistic physics. The relativity theory is studied after presentation of nonrelativistic physics. It is natural, that the relativity theory is presented in terms of nonrelativistic concepts. Such a presentation is clearer for physicists, which know nonrelativistic physics. New specific relativistic concepts are used only in the case, when one cannot ignore them.

However, the relativity theory is a self-sufficient fundamental theory, which can and must be presented without a mention of nonrelativistic concepts. *Furthermore, the relativity theory can be developed successfully only in terms of adequate (relativistic) concepts.*

Let there be two timelike world lines \mathcal{L}_1 and \mathcal{L}_2 of two different particles. Any point $P \in \mathcal{L}_1$ corresponds, at least, to one near point $P' \in \mathcal{L}_2$, i.e. $P' \simeq P$, because the timelike world line \mathcal{L}_2 crosses the light cone with the vertex at the point $P \in \mathcal{L}_1$. In other words, any point of the world line \mathcal{L}_1 has a near point on the world line \mathcal{L}_2 and vice versa.

Let us consider the space-time Ω of Minkowski, which is described by the world function σ_M , defined by (2.3). Let the inertial coordinate system K be used, and the world chains $\mathcal{C}_1, \mathcal{C}_2$ be timelike. The world chains \mathcal{C}_1 and \mathcal{C}_2 consist of connected segments $\mathcal{T}_{[P_l P_{l+1}]}$ and $\mathcal{T}_{[P'_l P'_{l+1}]}$

$$\mathcal{C}_1 = \bigcup_l \mathcal{T}_{[P_l P_{l+1}]}, \quad \mathcal{C}_2 = \bigcup_l \mathcal{T}_{[P'_l P'_{l+1}]} \quad (3.10)$$

$$\mathcal{T}_{[P_l P_{l+1}]} = \left\{ R \mid \sqrt{2\sigma_M(P_l, R)} + \sqrt{2\sigma_M(P_{l+1}, R)} = \sqrt{2\sigma_M(P_l, P_{l+1})} \right\} \quad (3.11)$$

$$\mathcal{T}_{[P'_l P'_{l+1}]} = \left\{ R \mid \sqrt{2\sigma_M(P'_l, R)} + \sqrt{2\sigma_M(P'_{l+1}, R)} = \sqrt{2\sigma_M(P'_l, P'_{l+1})} \right\} \quad (3.12)$$

All segments of a world chain have the same geometrical length μ , defined by the relation (3.3). The real mass of the particle, described by the world chain, is connected with the geometric mass μ by means of the relation (3.4).

Outside the microcosm the length μ is small with respect to characteristic size of the world chain, and one may consider, that the vectors $\mathbf{P}_l \mathbf{P}_{l+1}$ of any link have infinitesimal length. In the Minkowski space-time Ω the timelike links $\mathcal{T}_{[P_l P_{l+1}]}$ are one-dimensional infinitesimal timelike segments. The timelike world chain \mathcal{C} can be replaced by a smooth timelike world line \mathcal{L} , whose points are labelled by a parameter τ . The world line is described by the vector $\mathbf{P}\mathbf{P}'(\tau)$, where P is the origin of the coordinate system K and $P'(\tau) \in \mathcal{L}$. The vectors $\mathbf{P}_l \mathbf{P}_{l+1}$ of links turn into infinitesimal vectors $\mathbf{P}'(\tau) \mathbf{P}'(\tau + d\tau)$, which are tangent to the world line.

Let the world lines \mathcal{L}_1 and \mathcal{L}_2 be timelike. For timelike world lines the infinitesimal segments $\mathcal{T}_{[P_l P_{l+1}]} = \mathbf{P}'(\tau) \mathbf{P}'(\tau + d\tau)$ are timelike, and the geometrical mass μ is real ($\sigma(P_l, P_{l+1}) > 0$). In this case the world lines \mathcal{L}_1 and \mathcal{L}_2 are one-dimensional, and all points of a world line can be labelled by a parameter τ .

As far as the space-time Ω of Minkowski is a linear vector space, the vectors $\mathbf{PP}'(\tau)$ can be represented as a linear combination of basic vectors

$$\mathbf{PP}'(\tau) = f^k(\tau) \mathbf{e}_k, \quad (3.13)$$

where \mathbf{e}_k are basic vectors of the inertial coordinate system K with the origin P . The functions $f^k(\tau)$, $k = 0, 1, 2, 3$ are coordinates of points of the world line \mathcal{L}_2 .

Four basic vectors \mathbf{e}_k may be presented in the form

$$\mathbf{e}_k = \mathbf{PQ}_k, \quad \mathbf{e}^i = (g_M)^{ik} \mathbf{e}_k = (g_M)^{ik} \mathbf{PQ}_k, \quad k = 0, 1, 2, 3 \quad (3.14)$$

Here and further a summation over repeating Latin indices is produced $0 \div 3$. The basic vector $\mathbf{e}_k = \mathbf{PQ}_k$ is determined by the origin point P and by the end point Q_k . Such a representation is necessary to use the scalar product in arbitrary physical geometry, where there is no linear space, and the scalar product of two vectors \mathbf{PR} and $\mathbf{Q}_0\mathbf{Q}_1$ is defined by the relation (3.7). The scalar product $(\mathbf{PR} \cdot \mathbf{Q}_0\mathbf{Q}_1)$ of two vectors \mathbf{PR} and $\mathbf{Q}_0\mathbf{Q}_1$ is defined only via the world function without a reference to the properties of the linear vector space.

Coordinates of the points $P'(\tau)$ in the coordinate system K can be presented as follows

$$\mathcal{L}_2 : P'(\tau) = \{f^k(\tau)\} = \{(\mathbf{PP}'(\tau) \cdot \mathbf{e}^k)\} = \{(g_M)^{ik} (\mathbf{PP}'(\tau) \cdot \mathbf{e}_i)\}, \quad \tau \in \mathbb{R}, \quad P' \in \Omega \quad (3.15)$$

or

$$f^k(\tau) = (g_M)^{kl} (\mathbf{PP}'(\tau) \cdot \mathbf{e}_l) = (g_M)^{kl} (\mathbf{PP}'(\tau) \cdot \mathbf{PQ}_l) \quad (3.16)$$

$$f_k(\tau) = (g_M)_{kl} f^l(\tau) = (\mathbf{PP}'(\tau) \cdot \mathbf{PQ}_k) \quad (3.17)$$

where $(g_M)^{kl}$ is the contravariant metric tensor, which is obtained from the covariant metric tensor $(g_M)_{kl}$ by means of relations

$$(g_M)^{il} (g_M)_{lk} = \delta_k^i, \quad (g_M)_{lk} = (\mathbf{e}_i \cdot \mathbf{e}_k) = (\mathbf{PQ}_i \cdot \mathbf{PQ}_k) \quad (3.18)$$

In reality the functions $f^k(\tau)$ are piecewise. But for simplicity we shall consider them as continuous and differentiable

$$\mathcal{L}_2 : \quad x^k = f^k(\tau), \quad \dot{f}^k(\tau) \equiv \frac{df^k(\tau)}{d\tau} \quad k = 0, 1, 2, 3, \quad (3.19)$$

4 Dynamic equations for calculation of world function of space-time

Variation δg_{ik} of the metric tensor, which is generated by particles in the space-time geometry of Minkowski is described by the relation [10]

$$(c^{-2} \partial_0^2 - \nabla^2) \delta g_{ik} = -\kappa T_{ik} \quad (4.1)$$

where T_{ik} is the energy-momentum tensor of particles. The constant $\kappa = 8\pi G/c^2$, where G is the gravitational constant and c is the speed of the light. Solution of this equation can be presented in the form

$$\delta g_{ik}(x) = -\kappa \int G_{\text{ret}}(x, x') T_{ik}(x') \sqrt{-g_M} d^4 x', \quad (4.2)$$

$$g_M = \det ||(g_M)_{ik}||, \quad i, k = 0, 1, 2, 3 \quad (4.3)$$

where the retarded Green function $G_{\text{ret}}(x, x')$ has the form

$$G_{\text{ret}}(x, x') = \frac{1}{2\pi c} \theta(x^0 - x'^0) \delta(2\sigma_M(x, x')) \quad (4.4)$$

Here σ_M is the world function of the Minkowski space-time, defined by the relation (2.3), and the multiplier

$$\theta(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases} \quad (4.5)$$

The energy-momentum tensor T_{ik} of a particles has the form

$$T^{ik}(x) = \sum_s p_{(s)}^i(x) u_{(s)}^k(x), \quad i, k = 0, 1, 2, 3 \quad (4.6)$$

where $u_{(s)}^k(x)$ and $p_{(s)}^k(x)$ are distributions of the 4-velocity and of the 4-momentum of the s th particle in the space-time. We have for the particle number s

$$\mathcal{L}_{(s)} : \quad x^i = f_{(s)}^i(\tau), \quad p_{(s)}^i = b \dot{f}_{(s)}^i(\tau) d\tau, \quad g_{M(s)}^{ik} p_{(s)i} p_{(s)k} = m_{(s)}^2 c^2, \quad s = ..0, 1, \dots \quad (4.7)$$

$$u_{(s)}^k = \frac{\dot{f}_{(s)}^k(\tau)}{\sqrt{g_{rs} \dot{f}_{(s)}^r(\tau) \dot{f}_{(s)}^s(\tau)}} \quad (4.8)$$

$$p_{(s)i} = g_{Mik} b (\dot{f}_{(s)}^k(\tau + d\tau) - \dot{f}_{(s)}^k(\tau)) = g_{Mik} \dot{f}_{(s)}^k(\tau) b d\tau \quad (4.9)$$

where the constant b is the proportionality coefficient (3.4) between the length of the world line link $\mu = |\mathbf{P}_l \mathbf{P}_{l+1}|$ and the particle mass, described by this link. We have

$$p_{(s)i} g_{M(s)}^{ik} p_{(s)k} = g_{Mik} \dot{f}_{(s)}^i(\tau) \dot{f}_{(s)}^k(\tau) b^2 (d\tau)^2 = m_{(s)}^2 c^2 \quad (4.10)$$

$$m_{(s)} = \frac{b d\tau}{c} \sqrt{g_{M(s)} \dot{f}_{(s)}^r \dot{f}_{(s)}^s} \quad (4.11)$$

Then it follows from (4.7) and (4.11), that

$$p_{(s)}^i = \frac{m_{(s)} c \dot{f}_{(s)}^i}{\sqrt{g_{M(s)} \dot{f}_{(s)}^r \dot{f}_{(s)}^s}} \quad (4.12)$$

According (4.6) one obtains for the pointlike particles

$$T^{ik}(x) = \sum_s \frac{m_{(s)} c \dot{f}_{(s)}^i(\tau) \dot{f}_{(s)}^k(\tau)}{g_{M(s)} \dot{f}_{(s)}^r(\tau) \dot{f}_{(s)}^s(\tau)} \prod_{\alpha=1}^{\alpha=3} \delta_{\alpha}(x^{\alpha} - f_{(s)}^{\alpha}(\tau)) \quad (4.13)$$

where δ -function is defined by the relations

$$\int_V \prod_{\alpha=1}^{\alpha=3} F(\mathbf{x}') \delta_{\alpha}(x'^{\alpha} - f^{\alpha}(\tau)) \sqrt{-g_{\text{sp}}} d\mathbf{x}' = \begin{cases} F(\mathbf{f}(\tau)) & \text{if } \mathbf{x}' \in V \\ 0 & \text{if } \mathbf{x}' \notin V \end{cases} \quad (4.14)$$

Here

$$g_{\text{sp}} = \det \left\| (g_{\text{M}})_{\alpha\beta} \right\|, \quad \alpha, \beta = 1, 2, 3 \quad (4.15)$$

Integral (4.2) over

$$d^4 x' = d^3 \mathbf{x}' dt' = d^3 \mathbf{x}' \frac{dt'}{d\tau} d\tau = d^3 \mathbf{x}' \dot{f}^0(\tau) d\tau$$

can be presented in the form

$$\begin{aligned} \delta g^{ik}(x) &= -\kappa \int G_{\text{ret}}(x, x') T^{ik}(x') \sqrt{-g_{\text{M}}} d^4 x' \\ &= -\kappa \int \sum_s \frac{m_{(s)} \dot{f}_{(s)}^i(\tau) \dot{f}_{(s)}^k(\tau)}{2\pi g_{\text{M}rs} \dot{f}^r \dot{f}^s} \prod_{\alpha=1}^{\alpha=3} \delta_{\alpha}(x'^{\alpha} - f_{(s)}^{\alpha}(\tau)) \sqrt{-g_{\text{M}}} d^3 \mathbf{x}' \\ &\quad \times \delta(\sigma_{\text{M}}(x, f_{(s)}(\tau)) \dot{f}_{(s)}^0(\tau) d\tau \end{aligned} \quad (4.16)$$

Integration over \mathbf{x} gives

$$\delta g^{ik}(x) = -\kappa \int \sum_s \frac{m_{(s)} \dot{f}_{(s)}^i(\tau) \dot{f}_{(s)}^k(\tau)}{2\pi g_{\text{M}rs} \dot{f}^r(\tau) \dot{f}^s(\tau)} \sqrt{\frac{g_{\text{M}}}{g_{\text{sp}}}} \delta(2\sigma_{\text{M}}(x, f_{(s)}(\tau)) \dot{f}_{(s)}^0(\tau) d\tau \quad (4.17)$$

Integration over $d\tau$ gives

$$\delta g_{ik}(x) = -\frac{\kappa}{4\pi} \sum_s \frac{m_{(s)} g_{\text{M}ij} \dot{f}_{(s)}^j(\tau_s) g_{\text{M}kl} \dot{f}_{(s)}^l(\tau_s)}{g_{\text{M}rs} \dot{f}^r(\tau_s) \dot{f}^s(\tau_s) \left| \frac{d}{d\tau} \sigma_{\text{M}}(x, f(\tau_s)) \right|} \sqrt{\frac{g_{\text{M}}}{g_{\text{sp}}}} \dot{f}_{(s)}^0(\tau_s) \quad (4.18)$$

where $\tau_s = \tau_s(t, \mathbf{x})$ is a root of the equation

$$2\sigma_{\text{M}}(x, f(\tau_s)) = g_{\text{M}ik} (x^i - f_{(s)}^i(\tau_s)) (x^k - f_{(s)}^k(\tau_s)) = 0 \quad (4.19)$$

which can be written in the form

$$\sigma(P, P'_l) = 0 \quad (4.20)$$

We have

$$\frac{d}{d\tau} \sigma_{\text{M}}(x, f(\tau_s)) = -g_{\text{M}ik} (x^i - f_{(s)}^i(\tau_s)) \dot{f}^k(\tau_s) = -\frac{(\mathbf{P}\mathbf{P}'_l \cdot \mathbf{P}'_l \mathbf{P}'_{l+1})}{d\tau} \quad (4.21)$$

Using relations (3.16), (3.17) one can rewrite the relation (4.18) in the form

$$\delta g_{ik}(x) = -\frac{\kappa}{4\pi} \sum_s \frac{m_{(s)} (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P}\mathbf{Q}_i) (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P}\mathbf{Q}_k)}{(\mathbf{P}\mathbf{P}'_l \cdot \mathbf{P}'_l \mathbf{P}'_{l+1}) (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P}'_l \mathbf{P}'_{l+1})} (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P}\mathbf{Q}_k) g_{\text{M}}^{0k} \sqrt{\frac{g_{\text{M}}}{g_{\text{sp}}}} \quad (4.22)$$

In the case, when all basic vectors \mathbf{PQ}_k are unite and orthogonal, the determinants g_M and g_{sp} are connected by the relation

$$g_M = \det ||g_{Mik}|| = g_{sp} (g_M)_{00}, \quad (g_M)_{00} = (\mathbf{PQ}_0 \cdot \mathbf{PQ}_0) = |\mathbf{PQ}_0|^2 \quad (4.23)$$

The last multipliers of (4.22) can be written in the form

$$g_M^{00} \sqrt{\frac{g_M}{g_{sp}}} = ((g_M)_{00})^{-1} \sqrt{(g_M)_{00}} = \frac{1}{|\mathbf{PQ}_0|} \quad (4.24)$$

The constant κ is connected with the gravitational constant G by means of the relation $\kappa = 8\pi G/c^2$. Using (4.24) and (3.18), the relation (4.22) can be rewritten in terms of scalar products

$$\begin{aligned} \delta g_{ik}(P) &= \delta((\mathbf{PQ}_i \cdot \mathbf{PQ}_k)) \\ &= -\frac{2G}{c^2} \sum_s m_{(s)} \frac{\theta((\mathbf{P}'_l \mathbf{P} \cdot \mathbf{PQ}_0)) (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{PQ}_i) (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{PQ}_k) (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{PQ}_0)}{(\mathbf{P}'_l \mathbf{P} \cdot \mathbf{P}'_l \mathbf{P}'_{l+1}) (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P}'_l \mathbf{P}'_{l+1}) |\mathbf{PQ}_0|} \end{aligned} \quad (4.25)$$

$$\sigma(P, P'_l) = 0 \quad (4.26)$$

where vectors \mathbf{PQ}_i , $i = 0, 1, 2, 3$ are basic vectors of the coordinate system at the point P . Vector \mathbf{PQ}_0 is timelike. The points P'_l and P'_{l+1} are on the world line $\mathcal{L}_{(s)}$ of sth particle. The points P'_l and P'_{l+1} are separated by infinitesimal distance. All scalar products are taken in the space-time geometry of Minkowski. Besides, one uses the fact, that the metric tensor $g_{ik}(P)$ at the point P can be presented in the form

$$g_{ik}(P) = (\mathbf{PQ}_i \cdot \mathbf{PQ}_k), \quad i, k = 0, 1, 2, 3 \quad (4.27)$$

and the scalar product is expressed via the world function by means of the relation (3.7).

To determine world function σ from relations (4.25), (4.26), let us use the relation

$$(\mathbf{PS}_1 \cdot \mathbf{PS}_2) = \sigma(P, S_2) + \sigma(S_1, P) - \sigma(P, P) - \sigma(S_1, S_2) \quad (4.28)$$

As far as $\sigma(P, P) = 0$, it may rewritten in the form

$$\sigma(S_1, S_2) = \sigma(P, S_2) + \sigma(S_1, P) - (\mathbf{PS}_1 \cdot \mathbf{PS}_2) \quad (4.29)$$

Using (3.8) the relation (4.29) may be rewritten in terms of scalar products

$$\sigma(S_1, S_2) = \frac{1}{2} ((\mathbf{PS}_1 \cdot \mathbf{PS}_1) + (\mathbf{PS}_2 \cdot \mathbf{PS}_2) - 2(\mathbf{PS}_1 \cdot \mathbf{PS}_2)) \quad (4.30)$$

Replacing $Q_i \rightarrow S_1, Q_k \rightarrow S_2$ in relation (4.25) and substituting in (4.30), one obtains after transformations

$$\begin{aligned} \delta \sigma(S_1, S_2) &= -\frac{G}{c^2} \sum_s m_{(s)} \frac{\theta((\mathbf{P}'_l \mathbf{P} \cdot \mathbf{PQ}_0)) (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{PQ}_0)}{(\mathbf{P}'_l \mathbf{P} \cdot \mathbf{P}'_l \mathbf{P}'_{l+1}) |\mathbf{PQ}_0|} \\ &\quad \times \frac{((\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{PS}_1) - (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{PS}_2))^2}{(\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P}'_l \mathbf{P}'_{l+1})} \end{aligned} \quad (4.31)$$

The relations (4.31), (4.26) are completely geometric relations, written in terms of the world function σ_M of the Minkowski geometry. According to the deformation principle the relations (4.31), (4.26) are valid in any physical space-time geometry (i.e. for any world function σ). It means, that, if the space-time geometry without additional particles is described by the world function σ_0 , then appearance of additional particles perturbs the space-time geometry, and it becomes to be described by the world function $\sigma = \sigma_0 + \delta\sigma$, where perturbation $\delta\sigma$ of the world function is determined by the relations (4.31), (4.26). Scalar products in rhs of (4.31) should be calculated by means of the world function σ , which is unknown at first. As a result equations (4.31), (4.26) form equations for determination of the world function σ .

In the case of continuous distribution of particles the summation in (4.31) is to be substituted by integration over Lagrangian coordinates $\boldsymbol{\xi}$, labelling the perturbing particles. One obtains

$$\begin{aligned} \delta(S_1, S_2) = & -\frac{G}{c^2} \int_V \rho(\boldsymbol{\xi}) d\boldsymbol{\xi} \frac{\theta((\mathbf{P}'_l \mathbf{P} \cdot \mathbf{PQ}_0)) (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{PQ}_0)}{(\mathbf{P}'_l \mathbf{P} \cdot \mathbf{P}'_{l+1} \mathbf{P}) |\mathbf{PQ}_0|} \\ & \times \frac{((\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{PS}_1) - (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{PS}_2))^2}{(\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P}'_l \mathbf{P}'_{l+1})} \end{aligned} \quad (4.32)$$

where the total mass M is defined by the relation

$$\int_V \rho(\boldsymbol{\xi}) d\boldsymbol{\xi} = M \quad (4.33)$$

The points S_1 and S_2 are arbitrary points of the space-time. The point P is to be some function $P = P(S_1, S_2)$ of points S_1 and S_2 , which determine position of the point P between the points S_1 and S_2 . As far as $\delta\sigma(S_1, S_2)$ is a symmetric function of arguments S_1, S_2 , the function $P(S_1, S_2)$ is to be symmetric also, as it follows from (4.32)

$$P(S_1, S_2) = P(S_2, S_1) \quad (4.34)$$

The function $P(S_1, S_2)$ is to be determined by completely geometric relations, written in terms of the world function σ . In the geometry of Minkowski these relations can have the form

$$4\sigma(P, S_1) = \sigma(S_1, S_2), \quad 4\sigma(P, S_2) = \sigma(S_1, S_2) \quad (4.35)$$

Two equations (4.35) have an unique solution in the space-time of Minkowski, if the distance $S_1 S_2$ is timelike. The point P lies in the middle of the segment $S_1 S_2$. Indeed, let the world function $\sigma(S_1, S_2) > 0$, and the vector $\mathbf{S}_1 \mathbf{S}_2$ be timelike. Choosing coordinate system in such a way, that

$$S_1 = \{0, 0, 0, 0\} \quad S_2 = \{a_0, 0, 0, 0\}, \quad P = \{t, x^1, x^2, x^3\}, \quad (4.36)$$

the equations (4.35) are written in the form

$$c^2 t^2 - \mathbf{x}^2 = c^2 \frac{a_0^2}{4}, \quad c^2 (t - a_0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 = c^2 \frac{a^2}{4} \quad (4.37)$$

Taking difference of the two equations, one obtains

$$c^2 (2a_0 t - a_0^2) = 0, \quad t = \frac{a_0}{2} \quad (4.38)$$

Substituting $t = \frac{a_0}{2}$ in the second equation (4.37), one obtains

$$\mathbf{x}^2 = 0, \quad P = \left\{ \frac{a_0}{2}, 0, 0, 0 \right\} \quad (4.39)$$

If the segment $\mathbf{S}_1 \mathbf{S}_2$ is spacelike, the coordinate system may be chosen in such a way, that

$$S_1 = \{0, 0, 0, 0\} \quad S_2 = \{0, a_1, 0, 0\}, \quad P = \{t, x^1, x^2, x^3\} \quad (4.40)$$

In this case the solution of equations (4.35) looks as follows

$$P = \left\{ \sqrt{a_2^2 + a_3^2}, a_1, a_2, a_3 \right\} \quad (4.41)$$

where a_2 and a_3 are arbitrary quantities. Ambiguity of the solution is a manifestation of the Minkowski geometry multivariance with respect to spacelike vectors.

In this connection let us note, that at description of the particle motion only values of the world function for timelike intervals are important. Maybe, ambiguity of the world function for spacelike interval is not so important for description of the space-time properties. At any rate, one uses timelike world lines for description of particle dynamics.

5 World function of non-rotating body

Let us consider a physical body, which is concentrated in a space volume V . Its density is $\rho(\boldsymbol{\xi})$, where $\boldsymbol{\xi}$ are Lagrangian coordinates of the body points. The body does not rotate. We shall use the inertial coordinate system $x = \{t, \mathbf{x}\} = \{t, x^1, x^2, x^3\}$.

We shall search for solution of equations (4.32), (4.26) for the body in the form of a second order polynomial of $(t_1 - t_2)$

$$\sigma(t_1, \mathbf{y}_1; t_2, \mathbf{y}_2) = \frac{1}{2} A(\mathbf{y}_1, \mathbf{y}_2) c^2 (t_2 - t_1)^2 + B(\mathbf{y}_1, \mathbf{y}_2) c (t_2 - t_1) + C(\mathbf{y}_1, \mathbf{y}_2) \quad (5.1)$$

$$A(\mathbf{y}_1, \mathbf{y}_2) = 1 - V(\mathbf{y}_1, \mathbf{y}_2), \quad C(\mathbf{y}_1, \mathbf{y}_2) = -\frac{1}{2} (\mathbf{y}_1 - \mathbf{y}_2)^2 + \delta C(\mathbf{y}_1, \mathbf{y}_2) \quad (5.2)$$

where functions A, B and C should be determined as a result of solution of equations (4.32), (4.26). In the zeroth order approximation, when the space-time is the space of Minkowski, one has

$$A_0(\mathbf{y}_1, \mathbf{y}_2) = 1, \quad V_0(\mathbf{y}_1, \mathbf{y}_2) = 0, \quad B_0(\mathbf{y}_1, \mathbf{y}_2) = 0, \quad \delta C_0(\mathbf{y}_1, \mathbf{y}_2) = 0 \quad (5.3)$$

Let coordinates of points have the form

$$\begin{aligned}
P'_l &= \left\{ t - \frac{r}{c}, \boldsymbol{\xi} \right\}, & P'_{l+1} &= \left\{ t - \frac{r}{c} + dT, \boldsymbol{\xi} \right\}, \\
P &= \{t, \mathbf{x}\} & S_1 &= \{t_1, \mathbf{y}_1\} & S_2 &= \{t_2, \mathbf{y}_2\} \\
Q_0 &= \{t + dt, \mathbf{x}\}, & Q_1 &= \{t, x^1 + dx^1, x^2, x^3\}, \\
Q_2 &= \{t, x^1, x^2 + dx^2, x^3\}, & Q_3 &= \{t, x^1, x^2, x^3 + dx^3\}
\end{aligned} \tag{5.4}$$

where coordinates $\boldsymbol{\xi}$ label points of the body.

Vectors \mathbf{PQ} in scalar products of the expression (4.32) are described by coordinates of points P and Q : $\mathbf{PQ} = \{x(P); x(Q)\}$, where $x(P)$ are coordinates of the point P . By means of (5.4) we have the following coordinates for vectors in (4.32):

$$\begin{aligned}
\mathbf{P}'_l \mathbf{P} &= \left\{ t - \frac{r}{c}, \boldsymbol{\xi}; t, \mathbf{x} \right\}, & \mathbf{PQ}_0 &= \{t, \mathbf{x}; t + dt, \mathbf{x}\}, & \mathbf{P}'_l \mathbf{P}'_{l+1} &= \left\{ t - \frac{r}{c}, \boldsymbol{\xi}; t - \frac{r}{c} + dT, \boldsymbol{\xi} \right\}, \\
\mathbf{PS}_1 &= \{t, \mathbf{x}; t_1, \mathbf{y}_1\}, & \mathbf{PS}_2 &= \{t, \mathbf{x}; t_2, \mathbf{y}_2\}
\end{aligned} \tag{5.5}$$

The quantity dT is supposed to be infinitesimal.

In the first approximation the world function has the form

$$\sigma_1(t_1, \mathbf{y}_1; t_2, \mathbf{y}_2) = \frac{1}{2} A_1(\mathbf{y}_1, \mathbf{y}_2) c^2 (t_2 - t_1)^2 + B_1(\mathbf{y}_1, \mathbf{y}_2) c (t_2 - t_1) + C_1(\mathbf{y}_1, \mathbf{y}_2) \tag{5.6}$$

As far as $\sigma_1 = \sigma_M + \delta\sigma_1$, it follows from (5.6)

$$\delta\sigma_1(t_1, \mathbf{y}_1; t_2, \mathbf{y}_2) = -\frac{1}{2} V_1(\mathbf{y}_1, \mathbf{y}_2) c^2 (t_2 - t_1)^2 + B_1(\mathbf{y}_1, \mathbf{y}_2) c (t_2 - t_1) + \delta C_1(\mathbf{y}_1, \mathbf{y}_2) \tag{5.7}$$

where

$$\delta C_1(\mathbf{y}_1, \mathbf{y}_2) = C_1(\mathbf{y}_1, \mathbf{y}_2) + \frac{1}{2} (\mathbf{y}_2 - \mathbf{y}_1)^2 \tag{5.8}$$

At $t_2 \rightarrow t_1$ the world function $\sigma_1(t_1, \mathbf{y}_1; t_2, \mathbf{y}_1)$ tends to 0. Taking into account the symmetry of the world function with respect to transposition $(t_1, \mathbf{y}_1) \leftrightarrow (t_2, \mathbf{y}_1)$, we conclude, that

$$C_1(\mathbf{y}_1, \mathbf{y}_1) = 0, \quad B_1(\mathbf{y}_1, \mathbf{y}_1) = 0 \tag{5.9}$$

According to equation (4.26), we obtain from (5.6), (5.5)

$$\frac{1}{2} |\mathbf{P}'_l \mathbf{P}|^2 = \frac{1}{2} A_1(\boldsymbol{\xi}, \mathbf{x}) c^2 \left(\frac{r}{c} \right)^2 + B_1(\boldsymbol{\xi}, \mathbf{x}) r + C_1(\boldsymbol{\xi}, \mathbf{x}) = 0 \tag{5.10}$$

Solution of (5.10) has the form

$$\begin{aligned}
r &= \frac{-B_1(\boldsymbol{\xi}, \mathbf{x}) + \sqrt{B_1^2(\boldsymbol{\xi}, \mathbf{x}) - 2C_1(\boldsymbol{\xi}, \mathbf{x}) A_1(\boldsymbol{\xi}, \mathbf{x})}}{A_1(\boldsymbol{\xi}, \mathbf{x})} \\
&= -\frac{2C_1(\boldsymbol{\xi}, \mathbf{x})}{B_1(\boldsymbol{\xi}, \mathbf{x}) + \sqrt{B_1^2(\boldsymbol{\xi}, \mathbf{x}) - 2C_1(\boldsymbol{\xi}, \mathbf{x}) A_1(\boldsymbol{\xi}, \mathbf{x})}}
\end{aligned} \tag{5.11}$$

Calculation of other scalar products gives the results

$$|\mathbf{PQ}_0|^2 = A_1(\mathbf{y}_1, \mathbf{y}_2) c^2 (dt)^2 \quad (5.12)$$

$$(\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P}'_l \mathbf{P}'_{l+1}) = A_1(\boldsymbol{\xi}, \boldsymbol{\xi}) c^2 (dT)^2 \quad (5.13)$$

$$\begin{aligned} (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P}'_l \mathbf{P}) &= \sigma(P'_l, P) + \sigma(P'_{l+1}, P'_l) - 0 - \sigma(P'_{l+1}, P) \\ &= \sigma(P'_{l+1}, P'_l) - \sigma(P'_{l+1}, P) \end{aligned} \quad (5.14)$$

$$\begin{aligned} \sigma(P'_{l+1}, P) &= \sigma\left(t - \frac{r}{c} + dT, \boldsymbol{\xi}; t, \mathbf{x}\right) \\ &= \frac{1}{2} A_1(\boldsymbol{\xi}, \mathbf{x}) c^2 \left(\frac{r}{c} - dT\right)^2 + c B_1(\boldsymbol{\xi}, \mathbf{x}) \left(\frac{r}{c} - dT\right) + C_1(\boldsymbol{\xi}, \mathbf{x}) \end{aligned} \quad (5.15)$$

$$\begin{aligned} (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P}'_l \mathbf{P}) &= \sigma(P'_{l+1}, P'_l) - \sigma(P'_{l+1}, P) \\ &= \frac{1}{2} A_1(\boldsymbol{\xi}, \boldsymbol{\xi}) c^2 (dT)^2 - \left(\frac{1}{2} A_1(\boldsymbol{\xi}, \mathbf{x}) c^2 \left(\frac{r}{c} - dT\right)^2 + c B_1(\boldsymbol{\xi}, \mathbf{x}) \left(\frac{r}{c} - dT\right) + C_1(\boldsymbol{\xi}, \mathbf{x}) \right) \\ &= -\frac{1}{2} A_1(\boldsymbol{\xi}, \mathbf{x}) r^2 + A_1(\boldsymbol{\xi}, \mathbf{x}) c r dT - B_1(\boldsymbol{\xi}, \mathbf{x}) r + B_1(\boldsymbol{\xi}, \mathbf{x}) c dT - C_1(\boldsymbol{\xi}, \mathbf{x}) + \mathcal{O}(dT^2) \end{aligned} \quad (5.16)$$

Taking into account, the relation (5.10) one obtains

$$(\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P}'_l \mathbf{P}) =_1 (\boldsymbol{\xi}, \mathbf{x}) c r dT + B_1(\boldsymbol{\xi}, \mathbf{x}) c dT + \mathcal{O}(dT^2) \quad (5.17)$$

Calculation of $(\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P} \mathbf{S}_1) - (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P} \mathbf{S}_2)$ gives

$$\begin{aligned} &(\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P} \mathbf{S}_1) - (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P} \mathbf{S}_2) \\ &= \sigma_1(P'_l, S_1) + \sigma_1(P'_{l+1}, P) - \sigma_1(P'_l, P) - \sigma_1(P'_{l+1}, S_1) \\ &\quad - (\sigma_1(P'_l, S_2) + \sigma_1(P'_{l+1}, P) - \sigma_1(P'_l, P) - \sigma_1(P'_{l+1}, S_2)) \\ &= \sigma_1(P'_l, S_1) - \sigma_1(P'_{l+1}, S_1) - (\sigma_1(P'_l, S_2) - \sigma_1(P'_{l+1}, S_2)) \\ &= -dT \frac{\partial}{\partial dT} \sigma_1(P'_{l+1}, S_1) + dT \frac{\partial}{\partial dT} \sigma_1(P'_{l+1}, S_2) \\ &= dT \frac{\partial}{\partial dT} (\sigma_1(P'_{l+1}, S_2) - \sigma_1(P'_{l+1}, S_1)) + \mathcal{O}(dT^2) \end{aligned} \quad (5.18)$$

Using (5.1) and

$$\mathbf{P}'_l \mathbf{S}_1 = \left\{ t - \frac{r}{c}, \boldsymbol{\xi}; t_1, \mathbf{y}_1 \right\}, \quad \mathbf{P}'_{l+1} \mathbf{S}_1 = \left\{ t - \frac{r}{c} + dT, \boldsymbol{\xi}; t_1, \mathbf{y}_1 \right\} \quad (5.19)$$

one obtains from (5.18)

$$\begin{aligned} &(\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P} \mathbf{S}_1) - (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P} \mathbf{S}_2) \\ &= +A_1(\boldsymbol{\xi}, \mathbf{y}_1) c^2 t_1 dT - A_1(\boldsymbol{\xi}, \mathbf{y}_2) c^2 t_2 dT + (A_1(\boldsymbol{\xi}, \mathbf{y}_2) - A_1(\boldsymbol{\xi}, \mathbf{y}_1)) c^2 \left(t - \frac{r}{c}\right) dT \\ &\quad + (B_1(\boldsymbol{\xi}, \mathbf{y}_1) - B_1(\boldsymbol{\xi}, \mathbf{y}_2)) c dT + \mathcal{O}(dT^2) \end{aligned} \quad (5.20)$$

Let us take into account, that the time coordinate t of the point P has the form

$$t = \frac{t_1 + t_2}{2} \quad (5.21)$$

The relation (5.20) takes the form

$$\begin{aligned} & (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P} \mathbf{S}_1) - (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P} \mathbf{S}_2) \\ &= +\frac{1}{2} (A_1(\boldsymbol{\xi}, \mathbf{y}_1) + A_1(\boldsymbol{\xi}, \mathbf{y}_2)) c^2 (t_1 - t_2) dT - (A_1(\boldsymbol{\xi}, \mathbf{y}_2) - A_1(\boldsymbol{\xi}, \mathbf{y}_1)) r cdT \\ &+ (B_1(\boldsymbol{\xi}, \mathbf{y}_1) - B_1(\boldsymbol{\xi}, \mathbf{y}_2)) cdT \end{aligned} \quad (5.22)$$

Using (5.2), the relation (5.22) can be written in the form

$$\begin{aligned} & (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P} \mathbf{S}_1) - (\mathbf{P}'_l \mathbf{P}'_{l+1} \cdot \mathbf{P} \mathbf{S}_2) \\ &= \left(1 - \frac{1}{2} (V_1(\boldsymbol{\xi}, \mathbf{y}_2) + V_1(\boldsymbol{\xi}, \mathbf{y}_1)) \right) c^2 (t_1 - t_2) dT + (V_1(\boldsymbol{\xi}, \mathbf{y}_2) - V_1(\boldsymbol{\xi}, \mathbf{y}_1)) r cdT \\ &+ (B_1(\boldsymbol{\xi}, \mathbf{y}_1) - B_1(\boldsymbol{\xi}, \mathbf{y}_2)) cdT \end{aligned} \quad (5.23)$$

Calculation gives the following result for scalar product $(\mathbf{P}'_l \mathbf{P} \cdot \mathbf{P} \mathbf{Q}_0)$

$$(\mathbf{P}'_l \mathbf{P} \cdot \mathbf{P} \mathbf{Q}_0) = (2A_1(\boldsymbol{\xi}, \mathbf{x}) r + B_1(\boldsymbol{\xi}, \mathbf{x})) c dt \quad (5.24)$$

This scalar product is positive, if r , defined by the relation (5.11), is positive and $A_1(\boldsymbol{\xi}, \mathbf{x}) > 0$.

After substitution of expressions (5.9), (5.10), (5.13), (5.17) and (5.23), the expression (4.32) takes the form

$$\begin{aligned} & \delta \sigma(S_1, S_2) \\ &= -\frac{G}{c^2} \int_V \rho(\boldsymbol{\xi}) d\boldsymbol{\xi} \frac{\theta((\mathbf{P}'_l \mathbf{P} \cdot \mathbf{P} \mathbf{Q}_0)) A_1(\boldsymbol{\xi}, \mathbf{x}) c^2 dt dT}{\sqrt{A_1(\mathbf{x}, \mathbf{x})} cdT} \\ &\quad \times \frac{\left(\begin{aligned} & \left(1 - \frac{1}{2} (V_1(\boldsymbol{\xi}, \mathbf{y}_2) + V_1(\boldsymbol{\xi}, \mathbf{y}_1)) \right) c (t_1 - t_2) \\ & + (V_1(\boldsymbol{\xi}, \mathbf{y}_2) - V_1(\boldsymbol{\xi}, \mathbf{y}_1)) r + (B_1(\boldsymbol{\xi}, \mathbf{y}_1) - B_1(\boldsymbol{\xi}, \mathbf{y}_2)) \end{aligned} \right)^2 (cdT)^2}{(A_1(\boldsymbol{\xi}, \mathbf{x}) r + B_1(\boldsymbol{\xi}, \mathbf{x})) A_1(\boldsymbol{\xi}, \boldsymbol{\xi}) c^2 (dT)^2 cdT} \end{aligned} \quad (5.25)$$

After cancellation of multiplier dT and dt , we obtain

$$\begin{aligned} & \delta \sigma(S_1, S_2) \\ &= -\frac{G}{c^2} \int_V \rho(\boldsymbol{\xi}) d\boldsymbol{\xi} \frac{A_1(\boldsymbol{\xi}, \mathbf{x})}{\sqrt{A_1(\mathbf{x}, \mathbf{x})} (A_1(\boldsymbol{\xi}, \mathbf{x}) r + B_1(\boldsymbol{\xi}, \mathbf{x})) A_1(\boldsymbol{\xi}, \boldsymbol{\xi})} \\ &\quad \times \left(\begin{aligned} & \left(1 - \frac{1}{2} (V_1(\boldsymbol{\xi}, \mathbf{y}_2) + V_1(\boldsymbol{\xi}, \mathbf{y}_1)) \right) c (t_1 - t_2) \\ & + (V_1(\boldsymbol{\xi}, \mathbf{y}_2) - V_1(\boldsymbol{\xi}, \mathbf{y}_1)) r + (B_1(\boldsymbol{\xi}, \mathbf{y}_1) - B_1(\boldsymbol{\xi}, \mathbf{y}_2)) \end{aligned} \right)^2 \end{aligned} \quad (5.26)$$

where

$$r = \frac{-B_1(\boldsymbol{\xi}, \mathbf{x}) + \sqrt{B_1^2(\boldsymbol{\xi}, \mathbf{x}) - 2C_1(\boldsymbol{\xi}, \mathbf{x}) A_1(\boldsymbol{\xi}, \mathbf{x})}}{A_1(\boldsymbol{\xi}, \mathbf{x})} \quad (5.27)$$

One can see, that rhs of (5.26) is the second order polynomial of $(t_1 - t_2)$. Thus, our supposition that the world function is the second order polynomial of $(t_1 - t_2)$ is not changed after variation of the world function under influence of additional particles.

$$\delta\sigma_2(t_1, \mathbf{y}_1; t_2, \mathbf{y}_2) = -\frac{1}{2}V_2(\mathbf{y}_1, \mathbf{y}_2)c^2(t_2 - t_1)^2 + B_2(\mathbf{y}_1, \mathbf{y}_2)c(t_2 - t_1) + \delta C_2(\mathbf{y}_1, \mathbf{y}_2) \quad (5.28)$$

On the other side, it follows from (5.26)

$$\begin{aligned} & \delta\sigma_2(S_1, S_2) \\ = & -\int_V D(\mathbf{x}, \boldsymbol{\xi}) \left(1 - \frac{1}{2}(V_1(\boldsymbol{\xi}, \mathbf{y}_2) + V_1(\boldsymbol{\xi}, \mathbf{y}_1))\right)^2 c^2(t_1 - t_2)^2 d\boldsymbol{\xi} \\ & -2\int_V D(\mathbf{x}, \boldsymbol{\xi}) \left(1 - \frac{1}{2}(V_1(\boldsymbol{\xi}, \mathbf{y}_2) + V_1(\boldsymbol{\xi}, \mathbf{y}_1))\right) c(t_1 - t_2) \\ & \times ((V_1(\boldsymbol{\xi}, \mathbf{y}_2) - V_1(\boldsymbol{\xi}, \mathbf{y}_1))r + (B_1(\boldsymbol{\xi}, \mathbf{y}_1) - B_1(\boldsymbol{\xi}, \mathbf{y}_2))) d\boldsymbol{\xi} \\ & -\int_V D(\mathbf{x}, \boldsymbol{\xi}) (V_1(\boldsymbol{\xi}, \mathbf{y}_2) - V_1(\boldsymbol{\xi}, \mathbf{y}_1))r + (B_1(\boldsymbol{\xi}, \mathbf{y}_1) - B_1(\boldsymbol{\xi}, \mathbf{y}_2))^2 d\boldsymbol{\xi} \end{aligned} \quad (5.29)$$

where

$$\begin{aligned} D(\mathbf{x}, \boldsymbol{\xi}) &= \frac{G}{c^2} \frac{\rho(\boldsymbol{\xi}) A_1(\boldsymbol{\xi}, \mathbf{x})}{A_1(\boldsymbol{\xi}, \boldsymbol{\xi}) \sqrt{A_1(\mathbf{x}, \mathbf{x})} (A_1(\boldsymbol{\xi}, \mathbf{x})r + B_1(\boldsymbol{\xi}, \mathbf{x}))} \\ &= \frac{G}{c^2} \frac{\rho(\boldsymbol{\xi}) A_1(\boldsymbol{\xi}, \mathbf{x})}{A_1(\boldsymbol{\xi}, \boldsymbol{\xi}) \sqrt{A_1(\mathbf{x}, \mathbf{x})} \sqrt{B_1^2(\boldsymbol{\xi}, \mathbf{x}) - 2C_1(\boldsymbol{\xi}, \mathbf{x}) A_1(\boldsymbol{\xi}, \mathbf{x})}} \end{aligned} \quad (5.30)$$

Here

$$C_1(\boldsymbol{\xi}, \mathbf{x}) = -\frac{1}{2}(\mathbf{x} - \boldsymbol{\xi})^2 + \delta C_1(\boldsymbol{\xi}, \mathbf{x}) \quad (5.31)$$

Comparing (5.28) and (5.29), one concludes

$$V_2(\mathbf{y}_1, \mathbf{y}_2) = 2\int_V D(\mathbf{x}, \boldsymbol{\xi}) \left(1 - \frac{1}{2}(V_1(\boldsymbol{\xi}, \mathbf{y}_2) + V_1(\boldsymbol{\xi}, \mathbf{y}_1))\right)^2 d\boldsymbol{\xi} \quad (5.32)$$

$$\begin{aligned} B_2(\mathbf{y}_1, \mathbf{y}_2) &= -2\int_V D(\mathbf{x}, \boldsymbol{\xi}) \left(1 - \frac{1}{2}(V_1(\boldsymbol{\xi}, \mathbf{y}_2) + V_1(\boldsymbol{\xi}, \mathbf{y}_1))\right) \\ &\times ((V_1(\boldsymbol{\xi}, \mathbf{y}_2) - V_1(\boldsymbol{\xi}, \mathbf{y}_1))r + (B_1(\boldsymbol{\xi}, \mathbf{y}_1) - B_1(\boldsymbol{\xi}, \mathbf{y}_2))) d\boldsymbol{\xi} \end{aligned} \quad (5.33)$$

$$\delta C_2(\mathbf{y}_1, \mathbf{y}_2) = -\int_V D(\mathbf{x}, \boldsymbol{\xi}) (V_1(\boldsymbol{\xi}, \mathbf{y}_2) - V_1(\boldsymbol{\xi}, \mathbf{y}_1))r + (B_1(\boldsymbol{\xi}, \mathbf{y}_1) - B_1(\boldsymbol{\xi}, \mathbf{y}_2))^2 d\boldsymbol{\xi} \quad (5.34)$$

Substituting $V_2, B_2, \delta C_2$ in rhs of equations (5.32) - (5.34) instead of $V_1, B_1, \delta C_1$, we obtain the quantities $V_3, B_3, \delta C_3$. Continuing this process, we obtain in the limit,

that the quantities $V_n, B_n, \delta C_n$, appear to be equal in both sides of equations (5.32) - (5.34). In the developed form these equations are written as follows

$$V(\mathbf{y}_1, \mathbf{y}_2) = \frac{2G}{c^2} \int_V \frac{\rho(\boldsymbol{\xi}) A(\boldsymbol{\xi}, \mathbf{x}) \left(1 - \frac{1}{2} (V(\boldsymbol{\xi}, \mathbf{y}_2) + V(\boldsymbol{\xi}, \mathbf{y}_1))\right)^2}{A(\boldsymbol{\xi}, \boldsymbol{\xi}) \sqrt{A(\mathbf{x}, \mathbf{x})} \sqrt{B^2(\boldsymbol{\xi}, \mathbf{x}) + A(\boldsymbol{\xi}, \mathbf{x}) ((\mathbf{x} - \boldsymbol{\xi})^2 - 2\delta C(\boldsymbol{\xi}, \mathbf{x}))}} d\boldsymbol{\xi} \quad (5.35)$$

$$B(\mathbf{y}_1, \mathbf{y}_2) = -2 \frac{G}{c^2} \int_V \frac{\rho(\boldsymbol{\xi}) A(\boldsymbol{\xi}, \mathbf{x}) \left(1 - \frac{1}{2} (V(\boldsymbol{\xi}, \mathbf{y}_2) + V(\boldsymbol{\xi}, \mathbf{y}_1))\right)}{A_1(\boldsymbol{\xi}, \boldsymbol{\xi}) \sqrt{A(\mathbf{x}, \mathbf{x})} \sqrt{B^2(\boldsymbol{\xi}, \mathbf{x}) + A(\boldsymbol{\xi}, \mathbf{x}) ((\mathbf{x} - \boldsymbol{\xi})^2 - 2\delta C(\boldsymbol{\xi}, \mathbf{x}))}} d\boldsymbol{\xi} \\ \times ((V(\boldsymbol{\xi}, \mathbf{y}_2) - V(\boldsymbol{\xi}, \mathbf{y}_1)) r + (B(\boldsymbol{\xi}, \mathbf{y}_1) - B(\boldsymbol{\xi}, \mathbf{y}_2))) \quad (5.36)$$

$$\delta C(\mathbf{y}_1, \mathbf{y}_2) = -\frac{G}{c^2} \int_V \frac{\rho(\boldsymbol{\xi}) A(\boldsymbol{\xi}, \mathbf{x}) ((V(\boldsymbol{\xi}, \mathbf{y}_2) - V(\boldsymbol{\xi}, \mathbf{y}_1)) r + (B(\boldsymbol{\xi}, \mathbf{y}_1) - B(\boldsymbol{\xi}, \mathbf{y}_2)))^2}{A(\boldsymbol{\xi}, \boldsymbol{\xi}) \sqrt{A(\mathbf{x}, \mathbf{x})} \sqrt{B^2(\boldsymbol{\xi}, \mathbf{x}) + A(\boldsymbol{\xi}, \mathbf{x}) ((\mathbf{x} - \boldsymbol{\xi})^2 - 2\delta C(\boldsymbol{\xi}, \mathbf{x}))}} d\boldsymbol{\xi} \quad (5.37)$$

where

$$\mathbf{x} = \frac{\mathbf{y}_1 + \mathbf{y}_2}{2}, \quad A(\mathbf{y}_1, \mathbf{y}_2) = 1 - V(\mathbf{y}_1, \mathbf{y}_2) \quad (5.38)$$

$$r = \frac{-B(\boldsymbol{\xi}, \mathbf{x}) + \sqrt{B^2(\boldsymbol{\xi}, \mathbf{x}) + A(\boldsymbol{\xi}, \mathbf{x}) ((\mathbf{x} - \boldsymbol{\xi})^2 - 2\delta C(\boldsymbol{\xi}, \mathbf{x}))}}{A(\boldsymbol{\xi}, \mathbf{x})} \quad (5.39)$$

It follows from (5.36) - (5.37), that for $\mathbf{y}_1 = \mathbf{y}_2 = \mathbf{x}$

$$B(\mathbf{x}, \mathbf{x}) = 0, \quad \delta C(\mathbf{x}, \mathbf{x}) = 0 \quad (5.40)$$

Equations (5.35) - (5.37) are three integral equations for determination of three quantities $V(\mathbf{y}_1, \mathbf{y}_2)$, $B(\mathbf{y}_1, \mathbf{y}_2)$, $\delta C(\mathbf{y}_1, \mathbf{y}_2)$, which determine the world function

$$\sigma(t_1, \mathbf{y}_1; t_2, \mathbf{y}_2) = \frac{1}{2} c^2 (t_2 - t_1)^2 - \frac{1}{2} (\mathbf{y}_1 - \mathbf{y}_2)^2 - \frac{1}{2} V(\mathbf{y}_1, \mathbf{y}_2) c^2 (t_2 - t_1)^2 \\ + B(\mathbf{y}_1, \mathbf{y}_2) c (t_2 - t_1) + \delta C_1(\mathbf{y}_1, \mathbf{y}_2) \quad (5.41)$$

6 Dynamic equations for world function, generated by non-rotating sphere.

Let the shape of the physical body be a sphere of radius R . Let us introduce parameter $\varepsilon = r_g/R$, where $r_g = 2GM/c^2$ is so called gravitational radius. Let

$$\varepsilon = \frac{2G}{c^2} \int_V \frac{\rho(\boldsymbol{\xi})}{R} d\boldsymbol{\xi} \ll 1 \quad (6.1)$$

Then it follows from equations (5.37) - (5.39), that

$$V(\mathbf{y}_1, \mathbf{y}_2) = \mathcal{O}(\varepsilon), \quad B(\mathbf{y}_1, \mathbf{y}_2) = \mathcal{O}(\varepsilon^2), \quad \delta C(\mathbf{y}_1, \mathbf{y}_2) = \mathcal{O}(\varepsilon^3) \quad (6.2)$$

If $\varepsilon \ll 1$, equations (5.35) - (5.37) can be solved by means of successive approximations. In the first approximation one obtains

$$V_1(\mathbf{y}_1, \mathbf{y}_2) = \frac{2G}{c^2} \int_V \frac{\rho(\boldsymbol{\xi})}{\sqrt{\left(\frac{|\mathbf{y}_1 + \mathbf{y}_2|^2}{4} - \boldsymbol{\xi}\right)^2}} d\boldsymbol{\xi} + \mathcal{O}(\varepsilon^2) \quad (6.3)$$

$$B_1(\mathbf{y}_1, \mathbf{y}_2) = 0, \quad \delta C_1(\mathbf{y}_1, \mathbf{y}_2) = 0 \quad (6.4)$$

If $\rho(\boldsymbol{\xi})$

$$\rho(\boldsymbol{\xi}) = \begin{cases} \rho_0 & \text{if } |\boldsymbol{\xi}| < R \\ 0 & \text{if } |\boldsymbol{\xi}| > R \end{cases}, \quad \rho_0 = \frac{3M}{4\pi R^3} = \text{const} \quad (6.5)$$

where M is the sphere mass, then

$$V_1(\mathbf{y}_1, \mathbf{y}_2) = \begin{cases} \frac{2GM}{c^2|\mathbf{x}|} & \text{if } |\mathbf{x}| > R \\ 3\frac{GM}{c^2R} - \frac{GM}{c^2R^3}|\mathbf{x}|^2 & \text{if } |\mathbf{x}| < R \end{cases}, \quad \mathbf{x} = \frac{\mathbf{y}_1 + \mathbf{y}_2}{2} \quad (6.6)$$

In the second approximation one obtains

$$V_2(\mathbf{y}_1, \mathbf{y}_2) = \frac{2G}{c^2} \int_V \frac{\rho_0(\boldsymbol{\xi}) \sqrt{A_1(\boldsymbol{\xi}, \mathbf{x})} (1 - \frac{1}{2}(V_1(\boldsymbol{\xi}, \mathbf{y}_2) + V_1(\boldsymbol{\xi}, \mathbf{y}_1)))^2}{A_1(\boldsymbol{\xi}, \boldsymbol{\xi}) \sqrt{A_1(\mathbf{x}, \mathbf{x})} \sqrt{(\mathbf{x} - \boldsymbol{\xi})^2}} d\boldsymbol{\xi} + \mathcal{O}(\varepsilon^3) \quad (6.7)$$

$$\begin{aligned} B_2(\mathbf{y}_1, \mathbf{y}_2) &= -2\frac{G}{c^2} \int_V \frac{\rho_0(\boldsymbol{\xi}) \sqrt{A_1(\boldsymbol{\xi}, \mathbf{x})} (1 - \frac{1}{2}(V_1(\boldsymbol{\xi}, \mathbf{y}_2) + V_1(\boldsymbol{\xi}, \mathbf{y}_1)))}{A_1(\boldsymbol{\xi}, \boldsymbol{\xi}) \sqrt{A_1(\mathbf{x}, \mathbf{x})} \sqrt{(\mathbf{x} - \boldsymbol{\xi})^2}} d\boldsymbol{\xi} \\ &\times (V_1(\boldsymbol{\xi}, \mathbf{y}_2) - V_1(\boldsymbol{\xi}, \mathbf{y}_1)) r \end{aligned} \quad (6.8)$$

where

$$r = \frac{\sqrt{(\mathbf{x} - \boldsymbol{\xi})^2}}{\sqrt{A_1(\boldsymbol{\xi}, \mathbf{x})}}$$

$$B_2(\mathbf{y}_1, \mathbf{y}_2) = -2\frac{G}{c^2} \int_V \rho_0(\boldsymbol{\xi}) (V_1(\boldsymbol{\xi}, \mathbf{y}_2) - V_1(\boldsymbol{\xi}, \mathbf{y}_1)) d\boldsymbol{\xi} + \mathcal{O}(\varepsilon^3) \quad (6.9)$$

$$\begin{aligned} \delta C_2(\mathbf{y}_1, \mathbf{y}_2) &= -\frac{G}{c^2} \int_V \frac{\rho_0(\boldsymbol{\xi}) \sqrt{A_1(\boldsymbol{\xi}, \mathbf{x})} \left((V_1(\boldsymbol{\xi}, \mathbf{y}_2) - V_1(\boldsymbol{\xi}, \mathbf{y}_1)) \frac{\sqrt{(\mathbf{x} - \boldsymbol{\xi})^2}}{\sqrt{A_1(\boldsymbol{\xi}, \mathbf{x})}} \right)^2}{A_1(\boldsymbol{\xi}, \boldsymbol{\xi}) \sqrt{A_1(\mathbf{x}, \mathbf{x})} \sqrt{((\mathbf{x} - \boldsymbol{\xi})^2)}} d\boldsymbol{\xi} \\ &= -\frac{G}{c^2} \int_V \frac{\rho_0(\boldsymbol{\xi}) \sqrt{(\mathbf{x} - \boldsymbol{\xi})^2} (V_1(\boldsymbol{\xi}, \mathbf{y}_2) - V_1(\boldsymbol{\xi}, \mathbf{y}_1))^2}{A_1(\boldsymbol{\xi}, \boldsymbol{\xi}) \sqrt{A_1(\mathbf{x}, \mathbf{x})} A_1(\boldsymbol{\xi}, \mathbf{x})} d\boldsymbol{\xi} = \mathcal{O}(\varepsilon^3) \end{aligned} \quad (6.10)$$

We obtain

$$V_2(\mathbf{y}_1, \mathbf{y}_2) = \frac{2G}{c^2} \int_V \frac{\rho_0(\boldsymbol{\xi}) \sqrt{A_1(\boldsymbol{\xi}, \mathbf{x})} (1 - \frac{1}{2}(V_1(\boldsymbol{\xi}, \mathbf{y}_2) + V_1(\boldsymbol{\xi}, \mathbf{y}_1)))^2}{A_1(\boldsymbol{\xi}, \boldsymbol{\xi}) \sqrt{A_1(\mathbf{x}, \mathbf{x})} \sqrt{(\mathbf{x} - \boldsymbol{\xi})^2}} d\boldsymbol{\xi} + \mathcal{O}(\varepsilon^3) \quad (6.11)$$

$$\begin{aligned}
V_2(\mathbf{y}_1, \mathbf{y}_2) &= V_1(\mathbf{y}_1, \mathbf{y}_2) + \frac{G}{c^2} \int_V \frac{\rho_0(\boldsymbol{\xi}) (-V_1(\boldsymbol{\xi}, \mathbf{x}) + 2V_1(\boldsymbol{\xi}, \boldsymbol{\xi}) + V_1(\mathbf{x}, \mathbf{x}))}{\sqrt{(\mathbf{x} - \boldsymbol{\xi})^2}} d\boldsymbol{\xi} \\
&\quad - \frac{G}{c^2} \int_V \frac{\rho_0(\boldsymbol{\xi}) (V_1(\boldsymbol{\xi}, \mathbf{y}_2) + V_1(\boldsymbol{\xi}, \mathbf{y}_1))}{\sqrt{(\mathbf{x} - \boldsymbol{\xi})^2}} d\boldsymbol{\xi} + \mathcal{O}(\varepsilon^3)
\end{aligned} \tag{6.12}$$

$$\begin{aligned}
V_2(\mathbf{y}_1, \mathbf{y}_2) &= V_1(\mathbf{y}_1, \mathbf{y}_2) + \frac{2G}{c^2} \int_V \frac{\rho_0(\boldsymbol{\xi}) V_1(\boldsymbol{\xi}, \boldsymbol{\xi})}{\sqrt{(\mathbf{x} - \boldsymbol{\xi})^2}} d\boldsymbol{\xi} + \mathcal{O}(\varepsilon^3) \\
&\quad + \frac{G}{c^2} \int_V \frac{\rho_0(\boldsymbol{\xi}) (-V_1(\boldsymbol{\xi}, \mathbf{x}) + V_1(\mathbf{x}, \mathbf{x}) - V_1(\boldsymbol{\xi}, \mathbf{y}_2) - V_1(\boldsymbol{\xi}, \mathbf{y}_1))}{\sqrt{(\mathbf{x} - \boldsymbol{\xi})^2}} d\boldsymbol{\xi}
\end{aligned} \tag{6.13}$$

Estimation of (6.13) in the case, when $|\mathbf{y}_1|, |\mathbf{y}_2|, |\mathbf{x}| \gg R$, has the form

$$V_2(\mathbf{y}_1, \mathbf{y}_2) = V_1(\mathbf{y}_1, \mathbf{y}_2) + \frac{6}{5} \varepsilon^2 \frac{R}{|\mathbf{x}|} - \frac{\varepsilon^2 R^2}{2 |\mathbf{x}|^2} \left(1 + \frac{2|\mathbf{x}|}{|\mathbf{y}_1|} + \frac{2|\mathbf{x}|}{|\mathbf{y}_2|} \right) + \mathcal{O}(\varepsilon^3) \tag{6.14}$$

where $V_1(\mathbf{y}_1, \mathbf{y}_2)$ is determined by the relation (6.6), and

$$\varepsilon = \frac{2GM}{c^2 R} \ll 1 \tag{6.15}$$

In the case, when $\mathbf{y}_1 = \mathbf{y}_2 = \mathbf{x}$, we have

$$V_2(\mathbf{x}, \mathbf{x}) = V_1(\mathbf{x}, \mathbf{x}) + \frac{6}{5} \varepsilon^2 \frac{R}{|\mathbf{x}|} - \frac{5}{2} \varepsilon^2 \frac{R^2}{|\mathbf{x}|^2} + \mathcal{O}(\varepsilon^3) \tag{6.16}$$

We obtain for the quantity $B_2(\mathbf{y}_1, \mathbf{y}_2)$ for $|\mathbf{y}_2|, |\mathbf{y}_1| \gg R$

$$\begin{aligned}
B_2(\mathbf{y}_1, \mathbf{y}_2) &= -2 \frac{G}{c^2} \int_V \rho_0(\boldsymbol{\xi}) (V_1(\boldsymbol{\xi}, \mathbf{y}_2) - V_1(\boldsymbol{\xi}, \mathbf{y}_1)) d\boldsymbol{\xi} + \mathcal{O}(\varepsilon^3) \\
&= -2 \frac{GM}{c^2} (V_1(0, \mathbf{y}_2) - V_1(0, \mathbf{y}_1)) + \mathcal{O}(\varepsilon^3) \\
&= -\varepsilon^2 R^2 \left(\frac{1}{|\mathbf{y}_2|} - \frac{1}{|\mathbf{y}_1|} \right) + \mathcal{O}(\varepsilon^3)
\end{aligned} \tag{6.17}$$

$$B_2(\mathbf{x}, \mathbf{x}) = 0 \tag{6.18}$$

Thus, for small $\varepsilon = 2GM/(Rc^2)$ and $|\mathbf{x}| \gg R$, the calculated value of metric tensor, determined by the quantities $V_1(\mathbf{y}_1, \mathbf{y}_1), B_1(\mathbf{y}_1, \mathbf{y}_1), \delta C_1(\mathbf{y}_1, \mathbf{y}_1)$ coincides with the metric tensor, calculated in Newtonian approximation of the general relativity.

At large values of parameter ε the quantity $V(\mathbf{x}, \mathbf{x})$ remains to be less, than unity. Indeed, setting $\mathbf{y}_1 = \mathbf{y}_2 = \mathbf{x}$ in exact equations (5.35) - (5.37), we obtain

$$V(\mathbf{x}, \mathbf{x}) = \frac{2G}{c^2} \int_V \frac{\rho(\boldsymbol{\xi}) A(\boldsymbol{\xi}, \mathbf{x}) \left(1 - \frac{1}{2} (V(\boldsymbol{\xi}, \mathbf{x}) + V(\boldsymbol{\xi}, \mathbf{x})) \right)^2}{A(\boldsymbol{\xi}, \boldsymbol{\xi}) \sqrt{A(\mathbf{x}, \mathbf{x})} \sqrt{B^2(\boldsymbol{\xi}, \mathbf{x}) + A(\boldsymbol{\xi}, \mathbf{x}) ((\mathbf{x} - \boldsymbol{\xi})^2 - 2\delta C(\boldsymbol{\xi}, \mathbf{x}))}} d\boldsymbol{\xi} \tag{6.19}$$

$$B(\mathbf{x}, \mathbf{x}) = 0, \quad \delta C(\mathbf{x}, \mathbf{x}) = 0$$

Rewriting equation (6.19) in the form

$$\begin{aligned} & V(\mathbf{x}, \mathbf{x}) \sqrt{1 - V(\mathbf{x}, \mathbf{x})} \\ &= \frac{2G}{c^2} \int_V \frac{\rho(\xi) A(\xi, \mathbf{x}) \left(1 - \frac{1}{2}(V(\xi, \mathbf{x}) + V(\mathbf{x}, \xi))\right)^2}{A(\xi, \xi) \sqrt{B^2(\xi, \mathbf{x}) + A(\xi, \mathbf{x})(\mathbf{x} - \xi)^2 - 2A(\xi, \mathbf{x})\delta C(\xi, \mathbf{x})}} d\xi \end{aligned} \quad (6.20)$$

we conclude, that equation (6.20) contains only solutions with $V(\mathbf{x}, \mathbf{x}) \leq 1$. In other words, component $g_{00} = c^2(1 - V(\mathbf{x}, \mathbf{x}))$ of the metric tensor cannot change its sign. *It means that non-rotating physical body of any size and of any mass cannot generate a black hole.*

This result disagrees with the result of general relativity, but it agrees with the common sense. To obtain the reason of such unexpected result, we calculate the quantities $A, B, \delta C$ inside the uniform heavy sphere of radius R and mass M . At calculation we suppose that the quantity

$$\varepsilon = \frac{r_g}{R} = \frac{2GM}{c^2 R} \ll 1 \quad (6.21)$$

where r_g is the gravitational radius of the sphere.

For $|\mathbf{x}| < R$ results of calculations looks as follows (Details of calculations are rather bulky, and we omit them)

$$V_2(\mathbf{x}, \mathbf{x}) = \varepsilon \left(\frac{3}{2} - \frac{1}{2} \frac{\mathbf{x}^2}{R^2} \right) - \varepsilon^2 \frac{153}{64} + \varepsilon^2 \frac{37}{32} \frac{\mathbf{x}^2}{R^2} - \varepsilon^2 \frac{61}{320} \frac{|\mathbf{x}|^4}{R^4} + \mathcal{O}(\varepsilon^3) \quad (6.22)$$

The gravitational force inside the region $|\mathbf{x}| < R$ has the form

$$\mathbf{F} = \nabla V_2(\mathbf{x}, \mathbf{x}) = -\frac{\varepsilon}{R^2} \mathbf{x} + \frac{\varepsilon^2}{R^2} \frac{37}{16} \mathbf{x} - \frac{61}{80} \frac{\varepsilon^2}{R^2} \frac{|\mathbf{x}|^2}{R^2} \mathbf{x}, \quad |\mathbf{x}| < R \quad (6.23)$$

It follows from (6.23), that if $\varepsilon > \frac{16}{37} \approx 0.43$, the region, where the gravitational force is directed from the center, appears near the point $\mathbf{x} = 0$. If $\varepsilon \geq 0.65$, the gravitational force is directed from the center of the sphere in the whole region $|\mathbf{x}| < R$.

Thus, inside the heavy sphere the regions of antigravitation may appear at large values of ε . To understand this unexpected circumstance, let us note, that dynamical (not completely relativistic) approach and geometrical (completely relativistic) approach to gravitational phenomena disagree in some points.

The Newtonian gravitational potential of a uniform heavy sphere of radius R has the form

$$\varphi(\mathbf{x}) = \begin{cases} \frac{GM}{|\mathbf{x}|} & \text{if } |\mathbf{x}| > R \\ \frac{3GM}{2R} - \frac{GM}{2R^3} |\mathbf{x}|^2 & \text{if } |\mathbf{x}| < R \end{cases} \quad (6.24)$$

where M is the mass of the sphere. The gravitational potential φ is maximal at the point $\mathbf{x} = 0$, whereas the gravitational force $\mathbf{F} = \nabla \varphi$ is minimal at the point $\mathbf{x} = 0$ ($\mathbf{F} = 0$)

at $\mathbf{x} = 0$). The space-time geometry is connected with the gravitational potential $g_{00} = (c^2 - 2\varphi)$, but not with the gravitational force \mathbf{F} .

Gravitational potential φ inside the hollow sphere of mass M is proportional to the mass M , but $\varphi = \text{const}$, and the gravitational force $\mathbf{F} = 0$ inside the sphere. From dynamic (differential) viewpoint this fact is explained as a result compensation of gravitational influence of different parts of the hollow sphere. If the gravitational law distinguishes from the Newtonian one, such a compensation may disappear, and an induced antigravitation may appear, because the attraction force, generated by any part of the sphere, is directed from the center of the sphere.

From the geometric (integral) viewpoint an appearance of the induced antigravitation regions is natural, because the gravitational potential increases in such regions with increase of amount of the matter. As to the gravitational force, it may have any direction.

7 Concluding remarks

Thus, the extended general relativity (EGR) is the next stage of the physics geometrization. At this stage we have the monistic conception, containing only one fundamental quantity: world function σ . The gravitational field, which is one of fundamental quantities of the general relativity (GR), is now only an attribute of the world function. From viewpoint of extended general relativity the gravitational field is not a physical essence. It is only a manner of the particle interaction description. In particular, from viewpoint of EGR the gravitational field cannot exist separate from the matter. Such a change of approach to the gravitational field is connected with a usage of the relativistic concept of the events nearness.

Any monistic conception is a result of development of the preceding pluralistic conception, and the monistic conception is more perfect as a rule, than the preceding one. The extended general relativity (EGR) is obtained as a result of overcoming of defects of the general relativity (GR): (1) usage of only inconsistent Riemannian space-time geometry, (2) use of inadequate (nonrelativistic) concepts and quantities. EGR is to be considered as a more perfect conception, than GR. Results obtained in the framework of EGR are more dependable, than results, obtained in the framework of GR. In particular, conclusion on impossibility of the dark hole existence in EGR is more dependable, than existence of the black holes in the framework of GR. Besides, impossibility of the gravitational collapsing, leading to a formation of a black hole, is confirmed by appearance of induced antigravitation in EGR.

Besides, the mathematical technique of EGR is the same for all (continuous and discrete) geometries. Dynamic equations for the world function are written in the coordinateless form. This circumstance admits one to eliminate consideration of any coordinate transformation.

There is a possibility, that some problems of contemporary cosmology (dark energy, dark energy) are a result of imperfect theory of gravitation. More correct results of EGR, concerning dark holes, admit to hope, that EGR will be able to

solve difficult problems of contemporary cosmology.

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